First-order distributed Fermi acceleration of relativistic particles in nonuniform magnetic fields

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Topics:

1. Fokker-Planck transport equation
2. Diffusion approximation
3. Diffusion-convection transport equation
4. Application: Distributed first-order Fermi acceleration
5. Summary and conclusions

References:

Cosmic Ray Diffusion Approximation with Weak Adiabatic Focusing; Schlickeiser, R. & Shalchi, A., 2008, ApJ 686, 292

First-order distributed Fermi acceleration of relativistic particles in nonuniform magnetic fields with nonvanishing Alfvenic cross helicity turbulence; Schlickeiser, R., 2009, A & A, submitted

1. Fokker-Planck transport equation

The starting point for the transport of cosmic rays in magnetic field $\vec{B}_0(z) = B_0(z)\vec{e}_z$ with superposed weak electromagnetic turbulence $(\delta\vec{E}, \delta\vec{B})$ is the Fokker–Planck equation for the gyrotropic particle phase space density $f_0(X, Y, z, p, \mu, t)$ **per unit of magnetic line length**:

$$\frac{\partial f_0}{\partial t} + v\mu \frac{\partial f_0}{\partial z} - S_0(z,p,t) = \sum_{i,j} \frac{\partial}{\partial x_i} D_{x_i x_j} \frac{\partial f_0}{\partial x_j} - \frac{v}{2L}(1-\mu^2)\frac{\partial f_0}{\partial \mu} \tag{1}$$

where $x_i \in [\mu, p, X, Y]$ and

$$L^{-1}(z) = -\frac{1}{B_0}\frac{dB_0}{dz} \tag{2}$$

representing the adiabatic focusing of particles for spatial variations of the guide field $B_0(z)$. $L > 0$ for diverging guide field, $L < 0$ for converging guide field. For uniform fields $B_0 =$const. $\rightarrow L = \infty$. We restrict our analysis to isotropic source terms $S(z, p, t)$.

HERE: Consequences of additional adiabatic focusing term.

1.1. 2nd form of Fokker-Planck equation

The Fokker-Planck equation (1) holds for the gyrotropic part of the phase space density per unit of magnetic line length f_0 which is related to standard gyrotropic part of the cosmic ray phase space density f by $f_0 = fB(z)$. With the focusing length (2) we infer immediately

$$\frac{\partial f_0}{\partial z} = \frac{\partial}{\partial z}\left(fB(z)\right) = \frac{dB(z)}{dz}f + B(z)\frac{\partial f}{\partial z} = B(z)\left[\frac{\partial f}{\partial z} - \frac{f}{L}\right] \tag{3}$$

Inserting Eq. (3) into Eq. (1) we obtain as 2nd form of the Fokker-Planck equation

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial z} - \frac{S_0(z,p,t)}{B(z)} = \sum_{i,j}\frac{\partial}{\partial x_i}D_{x_ix_j}\frac{\partial f}{\partial x_j} - \frac{v}{2L}(1-\mu^2)\frac{\partial f}{\partial \mu} + v\mu\frac{f}{L}$$

$$= \sum_{i,j}\frac{\partial}{\partial x_i}D_{x_ix_j}\frac{\partial f}{\partial x_j} - \frac{\partial}{\partial \mu}\left[\frac{(1-\mu^2)v}{2L}f\right] \tag{4}$$

Adiabatic focusing term conserves total number of particles (only redistribution in μ)!

2. Diffusion approximation

For low-frequency MHD plasma turbulence: $\delta\vec{E} << \delta\vec{B}$ so that

$$D_{\mu\mu} >> D_{\mu p}, D_{\mu X}, D_{pp} \tag{5}$$

Consequently, the gyrotropic distribution function $f_0(X, Y, z, p, \mu, t)$ due to the dominating pitch-angle diffusion adjusts very quickly to a quasi-equilibrium through pitch-angle diffusion which is close to the isotropic equilibrium distribution $F_0(X, Y, z, p, t)$ per unit of magnetic line length:

$$f_0(X, Y, z, p, \mu, t) = F_0(X, Y, z, p, t) + g_0(X, Y, z, p, \mu, t) \tag{6}$$

where

$$F_0(X, Y, z, p, t) = \frac{1}{2}\int_{-1}^{1} d\mu\, f_0(X, Y, z, p, \mu, t), \tag{7}$$

$$\int_{-1}^{1} d\mu\, g_0(X, Y, z, p, \mu, t) = 0 \tag{8}$$

and where anisotropy $|g_0| \ll F_0$.

Substituting Eq. (6) into Eq. (1), averaging over μ and using $|g_0| \ll F_0$ yields

$$\frac{\partial F_0}{\partial t} = -\frac{v}{2}\Big[\frac{1}{L} + \frac{\partial}{\partial z}\Big] \int_{-1}^{1} d\mu \mu g_0 + \; S_0(z,p,t) +$$
$$\frac{1}{2p^2}\frac{\partial}{\partial p} p^2 \Big[\int_{-1}^{1} d\mu D_{p\mu}\frac{\partial g_0}{\partial \mu} + (\int_{-1}^{1} d\mu D_{pp})\frac{\partial F_0}{\partial p} + \sum_{i=1,2}(\int_{-1}^{1} d\mu D_{pX_i})\frac{\partial F_0}{\partial X_i}\Big]$$
$$+\frac{1}{2}\sum_{i=1,2}\frac{\partial}{\partial X_i}\Big[\sum_{j=1,2}(\int_{-1}^{1} d\mu D_{X_iX_j})\frac{\partial F_0}{\partial X_j} + \int_{-1}^{1} d\mu D_{X_i\mu}\frac{\partial g_0}{\partial \mu} + (\int_{-1}^{1} d\mu D_{X_ip})\frac{\partial F_0}{\partial p}\Big] \tag{9}$$

From difference of Eqs (1) and (9) derive approximation for cosmic ray anisotropy

$$g_0(z,p,\mu,t) = \Big[\int_{-1}^{1} d\mu \frac{(1-\mu)(1-\mu^2)}{D_{\mu\mu}(\mu)} - 2\int_{-1}^{\mu} dx \frac{1-x^2}{D_{\mu\mu}(x)}\Big]\frac{v}{4}\frac{\partial F_0}{\partial z}$$
$$+\Big[\int_{-1}^{1} d\mu \frac{(1-\mu)D_{\mu p}(\mu)}{D_{\mu\mu}(\mu)} - 2\int_{-1}^{\mu} dx \frac{D_{\mu p}(x)}{D_{\mu\mu}(x)}\Big]\frac{1}{2}\frac{\partial F_0}{\partial p}$$
$$+\sum_{i=1,2}\Big[\int_{-1}^{1} d\mu \frac{(1-\mu)D_{\mu X_i}(\mu)}{D_{\mu\mu}(\mu)} - 2\int_{-1}^{\mu} dx \frac{D_{\mu X_i}(x)}{D_{\mu\mu}(x)}\Big]\frac{1}{2}\frac{\partial F_0}{\partial X_i} \tag{10}$$

in order to calculate he integrals $\int_{-1}^{1} d\mu D_{p\mu}\frac{\partial g_0}{\partial \mu}$, $\int_{-1}^{1} d\mu D_{X_i\mu}\frac{\partial g_0}{\partial \mu}$ and $\int_{-1}^{1} d\mu \mu g_0$ in Eq. (9). Eq. (9) then becomes the diffusion-convection transport equation.

3. Diffusion-convection transport equation

Diffusion-convection equation for the isotropic part of the cosmic ray phase space distribution per unit of magnetic line length in the weak ($|L| >> \lambda$) adiabatic focusing limit is

$$\frac{\partial F_0}{\partial t} - S_0(z,p,t) =$$

$$\begin{pmatrix} \frac{\partial}{\partial X} \\ \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} \kappa_{XX} & \kappa_{XY} & -\kappa_{zX} \\ \kappa_{YX} & \kappa_{YY} & -\kappa_{zY} \\ \kappa_{zX} & \kappa_{zY} & \kappa_{zz} \end{pmatrix} \begin{pmatrix} \frac{\partial F_0}{\partial X} \\ \frac{\partial F_0}{\partial Y} \\ \frac{\partial F_0}{\partial z} \end{pmatrix} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 A \frac{\partial F_0}{\partial p}\right)$$

$$+\frac{v}{4}\frac{\partial}{\partial z}\left(a_{11}\frac{\partial F_0}{\partial p}\right) - \frac{1}{4p^2}\frac{\partial}{\partial p}\left(p^2 v a_{12}\frac{\partial F_0}{\partial z}\right) + \sum_{i=1,2}\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2 a_{21}\frac{\partial F_0}{\partial X_i} + \frac{\partial}{\partial X_i}a_{22}\frac{\partial F_0}{\partial p}\right)$$

$$+\frac{\kappa_{zz}}{L}\frac{\partial F_0}{\partial z} + \sum_{i=1,2}\frac{\kappa_{zi}}{L}\frac{\partial F_0}{\partial X_i} + \frac{v}{4}\frac{a_{11}}{L}\frac{\partial F_0}{\partial p} \tag{11}$$

with the pitch-angle averaged transport parameters

$$\kappa_{zz} = \frac{v\lambda}{3} = \frac{v^2}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}, \tag{12}$$

$$\kappa_{ij} = \frac{1}{2}\int_{-1}^{1} d\mu [D_{X_iX_j} - \frac{D_{X_i\mu}D_{\mu X_j}}{D_{\mu\mu}(\mu)}], \tag{13}$$

$$\kappa_{zi} = \frac{v}{4}\int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{X_i\mu}}{D_{\mu\mu}(\mu)}, \tag{14}$$

$$A = \frac{1}{2}\int_{-1}^{1} d\mu \Big[D_{pp}(\mu) - \frac{D_{\mu p}(\mu)D_{p\mu}(\mu)}{D_{\mu\mu}(\mu)}\Big], \tag{15}$$

$$a_{11} = \int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{\mu p}(\mu)}{D_{\mu\mu}(\mu)}, \tag{16}$$

$$a_{12} = \int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{p\mu}(\mu)}{D_{\mu\mu}(\mu)}, \tag{17}$$

$$a_{21} = \frac{1}{2}\int_{-1}^{1} d\mu \Big[D_{pX_i}(\mu) - \frac{D_{p\mu}(\mu)D_{\mu X_i}}{D_{\mu\mu}(\mu)}\Big], \tag{18}$$

and

$$a_{22} = \frac{1}{2}\int_{-1}^{1} d\mu \Big[D_{X_ip}(\mu) - \frac{D_{\mu p}(\mu)D_{X_i\mu}}{D_{\mu\mu}(\mu)}\Big], \tag{19}$$

respectively.

3.1. New transport terms due to weak adiabatic focusing

Adiabatic focusing gives rise to the last three terms in Eq. (11) that represent convective transport terms parallel to the guide field, perpendicular to the guide field and in momentum space, respectively. In the limit $L \to \infty$ of negligible adiabatic focusing these three new terms vanish.
The convective term along the guide field has been derived before by Earl (1976) and Kunstmann (1979); the other two are new. The respective convective speeds depend on the ratio of the corresponding diffusion coefficients or adiabatic deceleration rate to the focusing length. We first comment on each of them:

- Because of the earlier assumption of weak focusing ($|L| \gg \lambda$) the new parallel convective speed $\kappa_{zz}/L = v\lambda/3L$ is much less than the individual cosmic ray speed v.
- The off-diagonal diffusion tensor elements κ_{zi} vanish in the limit of axisymmetric turbulence. Therefore the convective terms perpendicular to the guide field do not arise in axisymmetric turbulence.

- Particularly interesting is the new convection term in momentum space

$$\frac{va_{11}}{4L}\frac{\partial F}{\partial p} = \frac{V_A H}{3L} p \frac{\partial F}{\partial p}$$

This convection term even occurs in the absence of any bulk motion of the background plasma for suitable plasma wave turbulence. The underlying physical reason is the difference between the effective plasma wave velocity, which determines the cosmic ray bulk velocity (Bell 1978, Schlickeiser and Vainio 1999), and the gas velocity. Even if there is no gas velocity, the cosmic ray bulk speed may be finite if we have different intensities of forward and backward moving plasma waves in the system, i.e. a non-zero cross helicity state.

For positive values of the product $a_{11}L > 0$ it represents a continuous momentum loss term, whereas for negative values $a_{11}L < 0$ it represents a first-order Fermi-type acceleration term. The focusing length $L(z)$ is positive for a diverging guide magnetic field (see Eq. (2)) and negative for a converging guide field. On the other hand, the absolute value and the sign of the deceleration rate a_{11} depend sensitively on the cross helicity and magnetic helicity of the magnetic field turbulence.

3.2. Physics explanation of the new 1st-order Fermi acceleration

New 1st-order Fermi acceleration is closely related to two effects:
(1) within the adiabatic guiding-center approximation of the transport of charged particles (e. g. Boyd and Sanderson 1969, Rossi and Olbert 1970), the magnetic moment of charged particles is an adiabatic invariant

$$\mu_M = \frac{p_\perp^2}{2m\gamma B(z)} = \frac{pv(1-\mu^2)}{2B(z)} = \text{const.} \tag{20}$$

in a slowly varying guide magnetic field L $\gg r_g$, with the particles' gyroradius r_g and the focusing length $L^{-1} = -\frac{1}{B(z)}\frac{dB(z)}{dz}$;
(2) if the physical system contains magnetohydrodynamic plasma waves such as Alfven waves, whose magnetic field component is much larger than their electric field component, the quickest particle-wave interaction process is pitch-angle scattering, so that the charged particle distribution function is isotropised on a very short time scale $\tau_{\text{iso}} \ll L/v$. Averaging the magnetic moment (20) with respect to the cosine of pitch-angle μ then yields for the respective quantities at the two positions $z = 0$ and z

$$\frac{< pv >_z}{< pv >_0} = \frac{B(z)}{B(0)} = \exp(-z/L) \tag{21}$$

where in the last step we assume an exponentially varying guide magnetic field.

If the intensities of forward (with parallel phase speed $+V_A$) moving Alfven waves (I^+) and backward (with parallel phase speed $-V_A$) moving Alfven waves (I^-) is different, the resulting net cross helicity of Alfven waves $H = (I^+ - I^-)/(I^+ + I^-)$ results in a net convection speed $V_N = HV_A$ of charged particles as each Alfvenic wave mode isotropises the particles in its rest frame. As a consequence, the average particle position convects as $z = 0 + V_N t = HV_A t$ so that according to Eq. (21) the particle momentum

$$< pv >_z (t) =< pv >_0 \exp\left[-\frac{HLV_A}{L^2}t\right] \tag{22}$$

increases exponentially with time if $HL < 0$ or decreases exponentially with time if $HL < 0$. For relativistic particles ($v \simeq c$) Eq. (22) implies the momentum acceleration rate $\dot{p}/p = -HV_A/L$ which apart from a factor 3 agrees with the exact rate.

This novel distributed 1st order Fermi acceleration process operates in all cosmic sources with $HL < 0$.

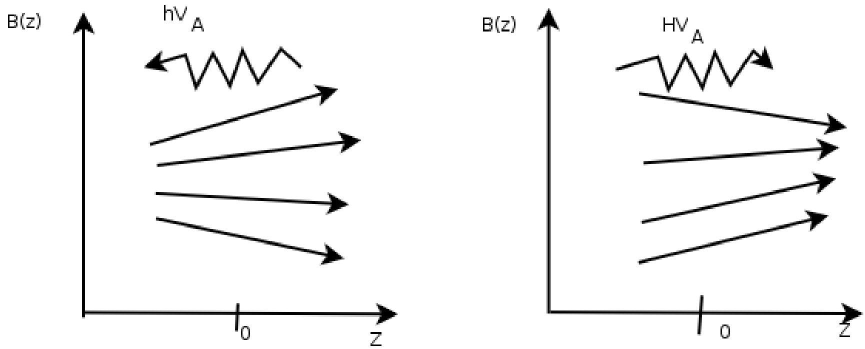

Figure 1: Conditions for 1st-order distributed Fermi acceleration in diverging (a) and converging (b) guide magnetic fields. In diverging magnetic fields a net negative ($h = -H_c < 0$) cross helicity state of Alfven waves (pronged curve) convects the average particle to regions of stronger field stength. In converging magnetic fields a net positive ($H_c > 0$) cross helicity state of Alfven waves also convects the average particle to regions of stronger field stength. In both cases the conservation of the pitch-angle averaged magnetic moment of the particle requires the increase of the particle momentum.

4. Application: Distributed 1st-order Fermi acceleration

The new 1st-order Fermi acceleration process operates at the two galactic sites: the upstream medium of shock waves and in haloes of spiral galaxies. In these sites the product $HL < 0$ is negative.

4.1. Cosmic shock waves

The anisotropic upstream precursor distribution of cosmic rays accelerated at shock waves damps the forward moving and amplifies the backward moving Alfven waves in the upstream medium of the shock wave. Forward and backward here refer to the wave propagation direction in the upstream medium rest frame. The turbulence observations near interplanetary shock waves (Bamert et al. 2004, 2008; Kallenbach et al. 2005) are fully consistent with the predicted (Vainio and Schlickeiser 1999) downstream amplification of magnetic field fluctuations when **purely backward** moving upstream waves interact with the shock wave. If the upstream guide magnetic field diverges, this implies $HL < 0$ so that the new extended first-order Fermi acceleration operates in whole upstream medium with a diverging guide magnetic field.
Then first-order Fermi acceleration of cosmic ray particles is spatially not only confined to the small shock transition region but operates in the larger upstream medium with a diverging guide magnetic field and predominantly backward moving Alfven waves.

4.1.1. Acceleration time scales

The shortest time scale for this additional first-order acceleration is $t_{\rm acc} = \frac{3L}{V_A}$. In order to assess its importance we compare with the standard diffusive shock acceleration time (Drury 1983)

$$t_{\rm sh} = \frac{\kappa_{zz}}{V_S^2} = \frac{\lambda v}{3M_A^2 V_A^2} = \frac{\lambda}{L}\frac{v}{V_A}\frac{t_{\rm acc}}{9M_A^2} \tag{23}$$

where we scale $V_S = M_A V_A$. For relativistic cosmic ray particles we obtain for the time scale ratio

$$\frac{t_{\rm acc}}{t_{\rm sh}} = 9M_A^2 \frac{V_A}{c}\frac{L}{\lambda} \tag{24}$$

With a typical value $\lambda/L \simeq 0.1$ we find in the interstellar medium ($V_A/c \simeq 2.3 \cdot 10^{-5}$)

$$\left(\frac{t_{\rm acc}}{t_{\rm sh}}\right)_{\rm interstellar} = \left(\frac{M_A}{22}\right)^2 \tag{25}$$

The additional first-order Fermi acceleration process is quicker than the standard diffusive shock acceleration process for shocks with Alfvenic Mach numbers smaller than 22 in the interstellar medium.

4.2. Haloes of galaxies

Radio continuum surveys of our own and external disk galaxies (for review see Sofue et al. 1986) reveal large-scale spatial variations of the galactic guide magnetic field perpendicular to the galactic plane suggesting the exponential variation $B(z) = B_0 \exp(-z/z_b)$ with values of z_b of a few hundred parsecs. . For the exponentially diverging galactic guide magnetic field the focusing length $L = z_b$ is positive and constant, Negative values of the Alfvenic corss helicity H_c will therefore lead to distributed first-order Fermi acceleration of cosmic rays provided that the acceleration rate dominates all continous momentum loss processes of cosmic ray particles.

In axisymmetric isospectral undamped slab Alfvenic turbulence with equal polarisation states of f- and b-Alfven waves the steady-state focused diffusion transport equation for $F_0 = FB(z)$ is

$$\kappa_{zz}\frac{\partial^2 F}{\partial z^2} - \left[\frac{\kappa_{zz}}{L} + V_A H\right]\frac{\partial F}{\partial z} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 A\frac{\partial F}{\partial p} + \frac{HV_A}{3L}p^3 F + p^2 \dot{p}_{\rm loss} F\right)$$

$$- \frac{F}{T_c} = -S(z,p) \qquad (26)$$

The transport terms involving the focusing length L result from the conventional guiding center equations of motion (Northrop 1963) which are valid for focusing length $\mathtt{L} \gg r_g$ much larger than the gyroradii r_g of cosmic ray particles which limits the cosmic ray particle rigidity to values

$$\frac{p}{\mathtt{Z}} \ll \frac{eB\mathtt{L}}{c} = 1.1 \cdot 10^{18} (B/4\ \mu\mathrm{G})(\mathtt{L}/300\ \mathrm{pc}) \quad \frac{eV}{c} \tag{27}$$

Note: for Fe-ions ($Z = 28$) rigidity upper limit corresponds to total energy limit

$$E_{\mathrm{tot,Fe}} \ll 3.0 \cdot 10^{19} (B/4\ \mu\mathrm{G})(\mathtt{L}/300\ \mathrm{pc}) \quad \mathrm{eV} \tag{28}$$

Applying the diffusion approximation to cosmic rays at energies above 10^{16} eV is justified by the small measured anisotropies at these energies (Antoni et al. 2004, Hörandel et al. 2006).

4.2.1. Conditions for distributed first-order Fermi acceleration

For negative cross helicity values $0 > H = -h$, $h > 0$, the term

$$\mathcal{L} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{HV_A}{3L}p^3F\right] = -\frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{hV_A}{3L}p^3F\right] \tag{29}$$

in the transport equation (26) describes first-order Fermi acceleration with the acceleration rate

$$\dot{p}_{\rm acc} = a_1 p, \quad a_1 = \frac{hV_A}{3L} \tag{30}$$

provided that this rate is larger than the particle's loss rate

$$\dot{p}_{\rm acc} > \dot{p}_{\rm loss} \tag{31}$$

Most of the interstellar medium is filled by the tenous coronal phase with ionised gas densities of about $n_{\rm ionised} \simeq 10^{-3} n_{-3}$ cm $^{-3}$. Scaling $B = 4b_4\,\mu$G implies $V_A \simeq 2.75 \cdot 10^7 b_4 n_{-3}^{-1/2}$ cm/s. Scaling $L = 300 L_{300}$ pc we obtain

$$a_1 = 3 \cdot 10^{-14}\frac{b_4 h}{L_{300} n_{-3}^{1/2}} \quad \mathrm{s}^{-1} \tag{32}$$

The acceleration condition (31) then becomes

$$a_1 > \frac{\dot{p}_{\rm loss}}{p} \tag{33}$$

4.2.2. Momentum loss rates

As shown by Lerche and Schlickeiser (1982) and Mannheim and Schlickeiser (1994) for galactic cosmic ray hadrons of charge Ze and mass number A the continuous momentum loss rate at momenta larger than 9 MeV$/c$ is dominated by ionisation, Coulomb and pion losses so that

$$\dot{p}_{\text{loss,hadrons}} \simeq n_{\text{gas}} \left[b_I Z^2 p^{-2} + b_\pi p \right] \tag{34}$$

with the total interstellar gas density n_{gas} and the constants $b_I = 1.5 \cdot 10^{11}$ (eV/c)^3 cm^3 s^{-1} and $b_\pi = 5.85 \cdot 10^{-16}$ cm^3 s^{-1}. At relativistic momenta the catastrophic loss time for hadrons heavier than protons due to spallation and fragmentation reactions is

$$T_c(A > 1) = 6 \cdot 10^{14} A^{-0.7} n_{\text{gas}}^{-1} \quad \text{s} \tag{35}$$

The continuous loss rate of relativistic electrons is

$$\dot{p}_{\text{loss,electrons}} \simeq n_{\text{gas}} \left[\delta_1 + \delta_2 p + \delta_3 p^2 \right] \tag{36}$$

with $\delta_1 = 3 \cdot 10^{-7}$ (eV/c) cm^3 s^{-1}, $\delta_2 = 8 \cdot 10^{-16}$ cm^3 s^{-1} and $\delta_3 = 10^{-25} f_B$ (eV/c)$^{-1}$ cm^3 s^{-1} with

$$f_B = \frac{\left(w_{\text{ph}} / 0.7 \text{ eV cm}^{-3} \right) + 0.11 (B / 4\ \mu\text{G})^2}{n_{\text{gas}}}$$

4.2.3. Hadrons

For hadrons $a_1 > b_\pi n_{\rm gas}$ is always larger than the pion production loss rate. The Coulomb and ionisation losses then define a threshold momentum value p_c, given by $a_1 = b_I n_{\rm gas}/p_c^3$ so that

$$p_c = \left(\frac{Z^2 b_I n_{\rm gas}}{a_1}\right)^{1/3} = 0.17 Z^{2/3} \left(\frac{n_{\rm gas} L_{300}}{b_4 h}\right)^{1/3} n_{-3}^{1/6} \ \frac{\mathrm{GeV}}{c} \qquad (37)$$

whose value depends only weakly on the assumed interstellar gas parameters. All hadrons with momenta above p_c will undergo this first-order acceleration process up to momenta determined by condition (27) which covers about 10 orders of magnitude.

4.2.4. Electrons

For electrons the acceleration condition (33) reads

$$G(p) < \frac{a_1 - \delta_2}{n_{\rm gas}} \simeq \frac{a_1}{n_{\rm gas}}, \quad G(p) = \frac{\delta_1}{p} + \delta_3 p \qquad (38)$$

The function $G(p)$ attains its minimum $G_{\rm min} = 2\sqrt{\delta_1 \delta_3} = 3.5 \cdot 10^{-16} f_B^{1/2}$ cm^3 s^{-1} at $p_e = (\delta_1/\delta_3)^{1/2} = 1.7 f_B^{-1/2}$ GeV/c which at typical parameter values is smaller than the left hand side of Eq. (38). First-order Fermi acceleration of electrons is restricted to the momentum range around p_e given by

$$p_{c1} \leq p \leq p_{c2} \tag{39}$$

with

$$p_{c1} \simeq \frac{\delta_1 n_{\rm gas}}{a_1} = 10^{-2} \frac{n_{\rm gas} L_{300} n_{-3}^{1/2}}{b_4 h} \quad \text{GeV}/c \tag{40}$$

and

$$p_{c2} \simeq \frac{a_1}{\delta_3 n_{\rm gas}} = 300 \frac{b_4 h}{n_{\rm gas} L_{300} n_{-3}^{1/2}} \quad \text{GeV}/c \tag{41}$$

Obviously, **we have identified a distributed first-order Fermi acceleration process that preferentially accelerates relativistic hadrons over ten orders of magnitude in momentum values**, whereas electrons are accelerated over 4 orders of magnitude in momentum values from 0.01 to 300 GeV/c.

5. Summary and conclusions

- The cosmic ray diffusion approximation in the weak adiabatic focusing limit gives rise to three new convective terms (parallel to the guide field, perpendicular to the guide field and in momentum space) in the diffusion-convection transport equation of cosmic rays. The respective convective speeds depend on the ratio of the corresponding diffusion coefficients or adiabatic deceleration rate to the focusing length.

- For positive values of the product $a_{11}L > 0$ the new momentum convection term represents a continuous momentum loss term, whereas for negative values $a_{11}L < 0$ it represents a first-order Fermi-type acceleration term. The focusing length $L(z)$ is positive for a diverging guide magnetic field and negative for a converging guide field. The absolute value and the sign of the deceleration rate a_{11} depend sensitively on the cross helicity and magnetic helicity of the magnetic field turbulence.

- The new 1st-order Fermi acceleration process operates at all cosmic sites where the product $H_cL < 0$ is negative. These include the upstream medium of shock waves and haloes of spiral galaxies.

- In spiral galaxies cosmic ray hadrons are preferentially accelerated over cosmic ray electrons which have larger continous momentum loss processes. Power law momentum spectra with and without exponential cut-offs are generated depending on the momentum variation of the parallel spatial diffusion coefficient.

- Consequences for kinetic turbulence models: it is essential to know the cross helicity state (different behaviour of forward and backward moving waves) of MHD turbulence in cosmic sources.